

Theory of hadronic B decays

Dan Pirjol

Center for Theoretical Physics, MIT
77 Massachusetts Avenue, Cambridge, MA 02139

I give an overview of the theory of hadronic nonleptonic B decays into two light mesons. Using the soft-collinear effective theory (SCET), a factorization theorem for these processes has been proven to leading order in $1/m_b$. The phenomenological implications of this factorization relation for $B \rightarrow \pi\pi$ decays are discussed, together with the prospects for determining α from these modes.

1 Introduction

The hadronic decays of B mesons provide a unique source of information about the flavor structure of the Standard Model. Due to the peculiar hierarchy structure of the CKM matrix, CP violation is an order unity effect in B decays. After several years of data taking from the B factories BABAR and BELLE, we are now in a position to perform precision tests of the CKM mechanism for CP violation.

It is therefore of considerable interest to have a better theoretical understanding of the hadronic dynamics of B decays. Two main approaches are widely followed: a) flavor symmetry methods [1, 2], where isospin or SU(3) flavor symmetry are used to reduce the number of independent hadronic amplitudes. b) dynamical approaches, based on the $1/m_b$ expansion and factorization theorems. Several such methods have been proposed and used extensively over the past few years, known as ‘QCD factorization’ (QCDF) [5] and ‘pQCD’ [6]. Recently, an effective theory approach based on the Soft-Collinear Effective Theory (SCET) [7] has been used to study these decays. The flavor symmetry approach is covered at this conference in the talk of J. Zupan [3] and some aspects of the second approach in the talk of H. Y. Cheng [4]. I will discuss here recent progress on hadronic decays using the SCET.

2 SCET factorization relation

The weak Hamiltonian mediating non-leptonic B decays is given by

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{f=d,s} \left[V_{ub}V_{uf}^* (C_1 O_1^u + C_2 O_2^u) + V_{cb}V_{cf}^* (C_1 O_1^c + C_2 O_2^c) - V_{tb}V_{tf}^* \sum_{i=3}^{10} C_i O_i^f \right],$$

where $f = d, s$ for $\Delta S = 0, 1$ transitions, respectively. The tree operators $O_{1,2}^q$ are defined as

$$O_1^q = (\bar{q}b)_{V-A}(\bar{f}q)_{V-A}, \quad O_2^q = (\bar{q}_\beta b_\alpha)_{V-A}(\bar{f}_\alpha q_\beta)_{V-A},$$

while O_{3-6} are the so-called QCD penguin operators, and O_{7-10} are the electroweak (EW) penguins. The Hamiltonian Eq. (1) is matched onto SCET_I at the scale $Q \sim m_b$

$$H_W = \frac{2G_F}{\sqrt{2}} \sum_{n,\bar{n}} \left\{ \sum_i \int [d\omega_j]_{j=1}^3 c_i(\omega_j) Q_i^{(0)}(\omega_j) + \sum_i \int [d\omega_j]_{j=1}^4 b_i(\omega_j) Q_i^{(1)}(\omega_j) + \mathcal{Q}_{c\bar{c}} + \dots \right\} (1)$$

where $c_i^{(f)}(\omega_j)$ and $b_i^{(f)}(\omega_j)$ are Wilson coefficients, the ellipses denote color-octet operators which do not contribute at leading order and higher order terms in Λ_{QCD}/Q , $Q = \{m_b, E\}$, and $\mathcal{Q}_{c\bar{c}}$ denotes operators containing a $c\bar{c}$ pair. Their precise form is not required in the following. We omit the dependence of the Wilson coefficients on the labels ω_j . The SCET operators appearing here are defined as ($q = u, d, s$)

$O(\lambda^0)$	$O(\lambda)$
$Q_1^{(0)} = [\bar{u}_{n,\omega_1} \not{P}_L b_v] [\bar{f}_{\bar{n},\omega_2} \not{P}_L u_{\bar{n},\omega_3}]$	$Q_1^{(1)} = \frac{-2}{m_b} [\bar{u}_{n,\omega_1} ig \not{\mathcal{B}}_{n,\omega_4}^\perp P_L b_v] [\bar{f}_{\bar{n},\omega_2} \not{P}_L u_{\bar{n},\omega_3}]$
$Q_{2,3}^{(0)} = [\bar{f}_{n,\omega_1} \not{P}_L b_v] [\bar{u}_{\bar{n},\omega_2} \not{P}_{L,R} u_{\bar{n},\omega_3}]$	$Q_{2,3}^{(1)} = \frac{-2}{m_b} [\bar{f}_{n,\omega_1} ig \not{\mathcal{B}}_{n,\omega_4}^\perp P_L b_v] [\bar{u}_{\bar{n},\omega_2} \not{P}_{L,R} u_{\bar{n},\omega_3}]$
$Q_4^{(0)} = [\bar{q}_{n,\omega_1} \not{P}_L b_v] [\bar{f}_{\bar{n},\omega_2} \not{P}_L q_{\bar{n},\omega_3}]$	$Q_4^{(1)} = \frac{-2}{m_b} [\bar{q}_{n,\omega_1} ig \not{\mathcal{B}}_{n,\omega_4}^\perp P_L b_v] [\bar{f}_{\bar{n},\omega_2} \not{P}_L q_{\bar{n},\omega_3}]$

where we have omitted operators which give rise to flavor-singlet light mesons. The operators Q_3 receive contributions only from electroweak penguins. The effective theory operators contain collinear fields along both n and \bar{n} directions [8].

It is convenient to write the Wilson coefficients of the SCET operators in a form which separates the contributions from different operators in the full theory Hamiltonian

$$c_i = \lambda_u^{(f)} c_{iu} + \lambda_t^{(f)} [c_{it}^p + c_{it}^{\text{ew}}], \quad b_i = \lambda_u^{(f)} c_{iu} + \lambda_t^{(f)} [b_{it}^p + b_{it}^{\text{ew}}]. \quad (2)$$

The Wilson coefficients of the $O(\lambda^0)$ operators are known to $O(\alpha_s(Q))$ [5], but those of the $O(\lambda^0)$ operators only to tree level. The dominant contributions of the EWPs to the SCET Wilson coefficients c_{it}^{ew} and b_{it}^{ew} come from $Q_{9,10}$ and are fixed by SU(3) symmetry to all orders in terms of the coefficients of the tree operators $c_{1,2u}$ and $b_{1,2u}$ [12].

The nonleptonic B decay amplitudes are obtained by taking the matrix elements of the SCET effective Hamiltonian Eq. (1) with subleading terms in the usoft-collinear Lagrangian [13]. The procedure is completely analogous to that followed in deriving factorization relations for the heavy-to-light form factors [10]. The main result for the $\bar{B} \rightarrow M_n M_{\bar{n}}$ nonleptonic decay amplitude at leading order in Λ/m_b can be written in a schematic form as

$$A = c(u) \star \phi_{\bar{n}}(u) \zeta^{BM_n} + b(x, z, u) \star \phi_n(x) \star \phi_{\bar{n}}(u) \star J(x, z, k_+) \star \phi_B(k_+) + (n \leftrightarrow \bar{n}) + \langle \mathcal{Q}_{c\bar{c}} \rangle \quad (3)$$

with c, b Wilson coefficients, $J(x, z, k_+)$ a jet function, ζ^{BM_n} is a nonperturbative soft function and $\phi_n(x), \phi_{\bar{n}}(u), \phi_B(k_+)$ are light-cone wavefunctions for the light mesons and the B meson respectively. The corrections to this formula are suppressed by one power of Λ/m_b .

The main features of this factorization formula are:

- The soft function ζ^{BM} is the same as that appearing in the heavy-to-light form factor $B \rightarrow M$ at large recoil [10].

- Jet universality. The jet function is the same as that entering the factorization relation for $B \rightarrow P, V_{\parallel}$ form factors.

These points show an unexpected relation between semileptonic and nonleptonic decays. This connection can be made more transparent by defining a new nonperturbative amplitude $\zeta_J^{BM_n}(u, z) \equiv \phi_n(x) \star J(x, z, k_+) \star \phi_B(k_+)$, which has the same scaling in $1/m_b$ as ζ^{BM_n} . In the following we will take as independent nonperturbative parameters $\zeta, \zeta_J(u, z)$, which effectively includes perturbative corrections at the collinear scale $\mu^2 = Q\Lambda$ to all orders.

3 $B \rightarrow \pi\pi$ decays

As an application of the formalism described above we discuss the nonleptonic $B \rightarrow \pi\pi$ decays. The amplitudes can be written in a compact form as

$$\begin{aligned} A(\overline{B}^0 \rightarrow \pi^+\pi^-) &= \lambda_u^{(d)}(-T - P_u) + \lambda_c^{(d)}(-P_c) + \lambda_t^{(d)}(-P_t) \equiv \lambda_u^{(d)}T_c(1 + r_c e^{i\delta_c} e^{i\phi}) \\ \sqrt{2}A(\overline{B}^0 \rightarrow \pi^0\pi^0) &= \lambda_u^{(d)}(-C + P_u) + \lambda_c^{(d)}P_c + \lambda_t^{(d)}P_t \equiv \lambda_u^{(d)}T_n(1 + r_n e^{i\delta_n} e^{i\phi}) \\ \sqrt{2}A(B^- \rightarrow \pi^-\pi^0) &= \lambda_u^{(d)}(T + C) \end{aligned} \quad (4)$$

We neglected here small contributions from electroweak penguins, which can be included in a model-independent way using isospin symmetry. The amplitudes on the rhs are defined as $T_c = -T - P_u + P_t, T_n = -C + P_u - P_t$ and $\phi = \gamma$.

The $B \rightarrow \pi\pi$ data is shown in Table 1 [15]. This includes the branching ratios and the time-dependent CP violation parameters $S_{\pi\pi}, C_{\pi\pi}$ in $B^0(t) \rightarrow \pi^+\pi^-$. The relevant branching ratio information is contained in the two ratios

$$R_c = \frac{Br(B^0 \rightarrow \pi^+\pi^-)}{2Br(B^- \rightarrow \pi^0\pi^-)} \frac{\tau_{B^+}}{\tau_{B^0}} = 0.445 \pm 0.062, \quad R_n = \frac{Br(B^0 \rightarrow \pi^0\pi^0)}{Br(B^- \rightarrow \pi^0\pi^-)} \frac{\tau_{B^+}}{\tau_{B^0}} = 0.292 \pm 0.063 \quad (5)$$

We show in Table I also $C_{\pi^0\pi^0}$, the direct CP asymmetry in $B^0 \rightarrow \pi^0\pi^0$, which was recently measured by the Babar and BELLE Collaborations.

3.1 Isospin analysis

For a given γ , the data on $S_{\pi\pi}, C_{\pi\pi}, R_c, R_n$ allows the determination of the amplitude parameters T, C, P in Eqs. (4). Adding in also $C_{\pi^0\pi^0}$, the weak phase $\alpha = \pi - \beta - \gamma$ can be determined with a four-fold ambiguity. This is the well-known isospin analysis of Gronau and London [17]. We present first the isospin analysis for fixed γ , and then compare the results with the SCET predictions. We discuss the prospects for a γ (or α) determination in Sec. 4.

We will present the results of the isospin analysis in terms of the parameters (r_c, δ_c, u, v) , where $t = (u, v)$ are the coordinates of the apex of the triangle of isospin amplitudes $1 + t_n = t$. We defined here $t = T/T_c, t_n = T_n/T_c$. The measurable parameters are given by

$$S_{\pi\pi} = \frac{-\sin(2\beta + 2\gamma) - 2r_c \cos \delta_c \sin(2\beta + \gamma) - r_c^2 \sin 2\beta}{1 + 2r_c \cos \delta_c \cos \gamma + r_c^2} \quad (6)$$

	BABAR	Belle	Avg.
$Br(B^+ \rightarrow \pi^+\pi^0)$	$5.8 \pm 0.6 \pm 0.4$	$5.0 \pm 1.2 \pm 0.5$	5.61 ± 0.61
$Br(B^0 \rightarrow \pi^0\pi^0)$	$1.17 \pm 0.32 \pm 0.10$	$2.32^{+0.44+0.22}_{-0.48-0.18}$	1.51 ± 0.28
$Br(B^+ \rightarrow \pi^+\pi^-)$	$4.7 \pm 0.6 \pm 0.2$	$4.4 \pm 0.6 \pm 0.3$	4.6 ± 0.4
$S_{\pi\pi}$	$-0.30 \pm 0.17 \pm 0.03$	-1.00 ± 0.22	-0.608 ± 0.135
$C_{\pi\pi}$	$-0.09 \pm 0.15 \pm 0.04$	-0.58 ± 0.17	-0.368 ± 0.112
$C_{\pi^0\pi^0}$	$-0.12 \pm 0.56 \pm 0.06$	$-0.43 \pm 0.51^{0.16}_{-0.17}$	-0.28 ± 0.39

Table 1: Data on $B \rightarrow \pi\pi$ decays after ICHEP 2004, Beijing [14, 15]. The CP-averaged branching ratios are quoted in units of 10^{-6} .

γ	(r_c, δ_c)	(u, v)	$C_{\pi^0\pi^0}$
54°	$(0.32 \pm 0.11, -1.12 \pm 0.40)$	$(1.36 \pm 0.21, -1.00 \pm 0.17)$ $(1.60 \pm 0.22, 0.53 \pm 0.21)$	-0.07 ± 0.29 0.48 ± 0.21
64°	$(0.49 \pm 0.14, -0.71 \pm 0.27)$	$(1.63 \pm 0.27, -0.92 \pm 0.21)$ $(1.80 \pm 0.27, 0.51 \pm 0.30)$	-0.24 ± 0.34 0.78 ± 0.21
74°	$(0.68 \pm 0.15, -0.53 \pm 0.20)$	$(1.95 \pm 0.41, -0.43 \pm 1.03)$ $(2.00 \pm 0.27, 0.11 \pm 1.12)$	0.12 ± 1.10 0.67 ± 0.84

Table 2: Amplitude parameters in $B \rightarrow \pi\pi$ for several input values for γ , together with the prediction for the CP asymmetry in $B \rightarrow \pi^0\pi^0$. For each value of γ there are two solutions for the tree amplitudes.

$$C_{\pi\pi} = \frac{2r_c \sin \delta_c \sin \gamma}{1 + 2r_c \cos \delta_c \cos \gamma + r_c^2} \quad (7)$$

$$R_c = \frac{1}{t^2}[1 + r_c^2 + 2r_c \cos \delta_c \cos \gamma], \quad R_n = \frac{1}{t^2}[t_n^2 + r_c^2 - 2r_c(\cos \delta_c(u-1) + \sin \delta_c v) \cos \gamma] \quad (8)$$

with $t^2 = u^2 + v^2$, $t_n^2 = (u-1)^2 + v^2$. The direct CP asymmetry in the $B^0 \rightarrow \pi^0\pi^0$ mode is

$$C_{\pi^0\pi^0} = -\frac{2[r_c \sin \delta_c(u-1) - r_c \cos \delta_c v] \sin \gamma}{t_n^2 - 2[r_c \cos \delta_c(u-1) + r_c \sin \delta_c v] \cos \gamma + r_c^2} \quad (9)$$

We show in Table 2 the results for the amplitude parameters extracted from the data corresponding to several values of γ . For each given value of γ , there are four solutions for the parameters (r_c, δ_c, u, v) , which fall into two sets with common values of (r_c, δ_c) . We select only the physical solution corresponding to $r_c \leq 1$, which gives the 2 solutions for the amplitudes T_c, T_n shown in Table 2. For each of these solutions we show also predictions for the direct CP asymmetry in the neutral pions channel $C_{\pi^0\pi^0}$. Similar analyses have been presented in [2, 18].

The absolute magnitudes of the amplitudes are set by $|T_c + T_n| = N_\pi(0.296 \pm 0.016)$ GeV, which can be extracted from $\text{Br}(B^- \rightarrow \pi^-\pi^0)$. [We denoted here $N_\pi = G_F/\sqrt{2}m_B^2 f_\pi$ and used $|V_{ub}| = 0.0039$.]

3.2 SCET analysis

The analysis discussed above used only isospin symmetry. Next we examine the predictions from the SCET factorization formula.

1. *Predicting $Br(B^0 \rightarrow \pi^0\pi^0)$.* At tree level in matching, the strong phases of the tree amplitudes vanish $\text{Arg}(T_n/T_c) \sim O(\alpha_s(Q), \Lambda/Q)$. This fixes one hadronic parameter ($v \rightarrow 0$),

such that R_c, R_n and $C_{\pi^0\pi^0}$ are not independent quantities, but are related as

$$R_n = \frac{1}{t}[(t-1)(1-tR_c) + r_c^2], \quad C_{\pi^0\pi^0} = -(t-1)\frac{R_c}{R_n}C_{\pi^+\pi^-} \quad (10)$$

This allows predictions for $R_n(\alpha)$ and $C_{\pi^0\pi^0}(\alpha)$ to be made using only data on $R_c, S_{\pi\pi}, C_{\pi\pi}$. We show in Fig. 1 (a) the prediction for $R_n(\alpha)$ from Eq. (10) as a function of α .

2. *Determining the SCET nonperturbative parameters.* The LO factorization relation for T and T_c expresses these amplitudes in terms of SCET Wilson coefficients and the nonperturbative parameters $\zeta, \zeta_J(x)$. Working at tree level in matching, one finds

$$\zeta^{B\pi}\big|_{\gamma=64^\circ} = (0.08 \pm 0.03) \left(\frac{3.9 \times 10^{-3}}{|V_{ub}|} \right), \quad \zeta_J^{B\pi}\big|_{\gamma=64^\circ} = (0.10 \pm 0.02) \left(\frac{3.9 \times 10^{-3}}{|V_{ub}|} \right), \quad (11)$$

which does not include any theoretical uncertainties. These values can be used to predict the $B \rightarrow \pi$ form factor $f_+(0)$ at $q^2 = 0$ as

$$f_+(0) = \zeta^{B\pi} + \zeta_J^{B\pi} = (0.18 \pm 0.05) \left(\frac{3.9 \times 10^{-3}}{|V_{ub}|} \right)$$

which is somewhat lower than recent results from QCD sum rules $f_+(0) = 0.26 \pm 0.03$ [25]. The result for $\zeta_J^{B\pi}$ in Eq. (11) is slightly higher than the values obtained in perturbation theory working at tree level in the jet function in Eq. (3)

$$\zeta_J^{B\pi} = \frac{\pi\alpha_s C_F}{N_c} \frac{f_B f_\pi}{m_B} \langle \bar{x}^{-1} \rangle_\pi \langle k_+^{-1} \rangle_B \sim 0.02 - 0.05 \quad (12)$$

where we took $\langle \bar{x}^{-1} \rangle_\pi = 3(1 + a_2^\pi)$ with $a_2^\pi = 0.2 \pm 0.2$, $f_B = 200$ MeV, $f_\pi = 131$ MeV and $\langle k_+^{-1} \rangle_B = 1/\lambda_B$ with $\lambda_B = (350 \pm 150)$ MeV [5]. [The $O(\alpha_s^2(m_b\Lambda))$ corrections to this result have been recently obtained in [19].] Conceivable explanations for this discrepancy are experimental errors in the $B \rightarrow \pi\pi$ data, or neglected power corrections to the factorization formula. A detailed analysis using the QCDF approach [20] shows that power corrections are small in the tree amplitudes T, T_c and thus do not affect significantly this determination of $\zeta_J^{B\pi}$.

4 Prospects for determining α

The main motivation for the experimental study of the $\Delta S = 0$ decays such as $B \rightarrow \pi^+\pi^-$ is in connection with the determination of the angle α . In fact what is measured is the combination $\beta + \gamma = \pi - \alpha$, which taken together with the precise value of β known from charmonium modes, can be translated into a value of γ .

The measurements are usually expressed in terms of an effective angle α_{eff} defined by $\sin 2\alpha_{\text{eff}} = S_{\pi\pi}/\sqrt{1 - C_{\pi\pi}^2}$. This is related to the physical angle by $\alpha_{\text{eff}} = \alpha - \theta$ with $\theta = \text{Arg}(A_{\pi^+\pi^-}/\bar{A}_{\pi^+\pi^-})$. Using only data on R_c, R_n, C , only bounds on θ can be obtained. These bounds can be turned into a determination provided that $C_{\pi^0\pi^0}$ is also measured [17].

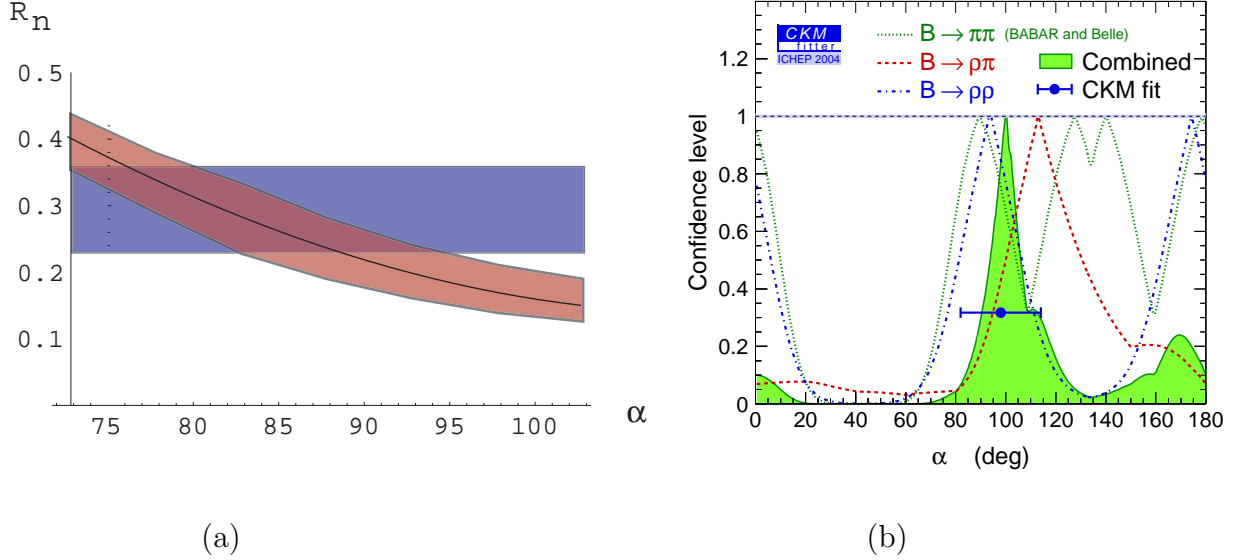


Figure 1: (a) Constraint on the weak phase α following from a small relative strong phase between the tree amplitudes T, T_c . The light band shows the 1σ prediction for $R_n(\alpha)$ as a function of the weak phase α following from Eq. (10); the horizontal band shows the measured value $R_n = 0.292 \pm 0.063$. The solid line denotes the GLSS bound $R_n \geq R_n^{GLSS}(\alpha, R_c, S_{\pi\pi}, C_{\pi\pi})$ for central values of the parameters. (b) constraints on α from charmless B decays [26].

Several bounds on θ using only isospin have been given in Refs. [21, 22], of which the most restrictive one is the GLSS bound [22]

$$\cos 2\theta \geq \frac{1}{2R_c\sqrt{1-C_{\pi\pi}^2}}[(R_c + 1 - R_n)^2 - 2R_c] \quad (13)$$

With the present data in Table I these bounds constrain α to lie within 4 windows of width 2θ with $\theta = 29.0^\circ$ centered on $\alpha_{\text{eff}} = 1/2 \arcsin(S_{\pi\pi}/\sqrt{1-C_{\pi\pi}^2})$. The window corresponding to the physically preferred solution has $\alpha_{\text{eff}}^I = 110.41^\circ$. These can be translated into bounds on $\beta + \gamma$ which gives $40.6^\circ = 69.6^\circ - \theta \leq \gamma + \beta \leq 69.6^\circ + \theta = 98.6^\circ$.

These bounds can be turned into a determination of α provided that $C_{\pi^0\pi^0}$ is known, which is equivalent to performing the full isospin analysis. In the absence of this information, one can restrict the ranges of the bound by adding in dynamical information about the amplitudes T, P, C . Several such “constrained” bounds exist, of which we mention only two.

- The Buchalla-Safir bound [24]. This assumes that $|\delta_c| \leq \pi/2$, which leads to a lower bound on $\overline{\eta}$ as a function of $S_{\pi\pi}$.
- The SCET constraint [11, 27]. The dynamical input here is the smallness of the relative strong phase of T_c and T_n .

$$\text{Arg}(T/T_c) \sim \text{Arg}(T_n/T_c) \sim O(\alpha_s(m_b), \Lambda/m_b) \quad (14)$$

We will discuss here in some detail the SCET constraint. We show in Fig. 1 (a) the constraints on α from comparing the prediction for $R_n(\alpha)$ from requiring a flat tree triangle Eq. (10) with the measured value of this parameter $R_n = 0.292 \pm 0.063$. One additional constraint is introduced by the GLSS bound (13) which bounds R_n from below $R_n \geq R_n^{GLSS}(\alpha, R_c, C_{\pi\pi})$. Taking into account all these constraints, the plot in Fig. 1 (a) allows the range for α

$$73^\circ \leq \alpha \leq 95^\circ \quad (15)$$

which includes only the experimental uncertainties. This result is in agreement with present constraints on α from $\Delta S = 0$ modes (see Fig. 1(b)) and with a general constraint combining $B \rightarrow \pi\pi, \rho\pi, \rho\rho$ modes [28].

A method for determining γ based on this constraint was proposed in Ref. [27], where it was argued that theoretical uncertainties to the condition Eq. (14) from radiative corrections and power corrections of canonical size introduce a very small theoretical error on α of $\pm 2^\circ$ (for the present central values of the data). More detailed theoretical computations of the correction to Eq. (14) would be welcome. With improved data the SCET constraint Eq. (14) can be expected to give useful information on α , complementing alternative determinations of this weak phase.

4.1 Acknowledgments

I would like to thank the organizers for an enjoyable conference. This work has been supported by the U.S. Department of Energy under cooperative research agreement DOE-FC02-94ER40818.

References

- [1] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D **50**, 4529 (1994); Phys. Rev. D **52**, 6374 (1995).
- [2] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, arXiv:hep-ph/0402112; A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Eur. Phys. J. C **32**, 45 (2003);
- [3] J. Zupan, *Determining α and γ - theory*, [arXiv:hep-ph/0410371].
- [4] H. Y. Cheng, *Direct CP violation and final state interactions in hadronic B decays*, [arXiv:hep-ph/0411340].
- [5] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. B **606**, 245 (2001); M. Beneke and M. Neubert, Nucl. Phys. B **675**, 333 (2003).
- [6] Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Rev. D **63**, 054008 (2001); C. H. Chen, Y. Y. Keum and H. n. Li, Phys. Rev. D **64**, 112002 (2001); H. Y. Cheng, C. K. Chua and A. Soni, arXiv:hep-ph/0409317.

- [7] C. W. Bauer, S. Fleming and M. E. Luke, Phys. Rev. D **63**, 014006 (2001); C. W. Bauer, S. Fleming, D. Pirjol and I. W. Stewart, Phys. Rev. D **63**, 114020 (2001); C. W. Bauer and I. W. Stewart, Phys. Lett. B **516**, 134 (2001); Phys. Rev. D **65**, 054022 (2002).
- [8] C. W. Bauer, S. Fleming, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D **66**, 014017 (2002).
- [9] J. g. Chay and C. Kim, Phys. Rev. D **68**, 071502 (2003); Nucl. Phys. B **680**, 302 (2004).
- [10] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D **67**, 071502 (2003); D. Pirjol and I. W. Stewart, Phys. Rev. D **67**, 094005 (2003) [Erratum-ibid. D **69**, 019903 (2004)]; D. Pirjol and I. W. Stewart, eConf **C030603**, MEC04 (2003) [arXiv:hep-ph/0309053].
- [11] C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D **70**, 054015 (2004).
- [12] C. W. Bauer and D. Pirjol, Phys. Lett. B **604**, 183 (2004)
- [13] M. Beneke, A. P. Chapovsky, M. Diehl and T. Feldmann, Nucl. Phys. B **643**, 431 (2002); M. Beneke and T. Feldmann, Phys. Lett. B **553**, 267 (2003)
- [14] The Heavy Flavor Averaging Group, <http://www.slac.stanford.edu/xorg/hfag/>
- [15] B. Aubert [BABAR Collaboration], arXiv:hep-ex/0412037; K. Abe *et al.* [Belle Collaboration], arXiv:hep-ex/0408101. B. Aubert [BABAR Collaboration], arXiv:hep-ex/0501071.
- [16] M. Neubert and J.L. Rosner, Phys. Lett. B **441**, 403 (1998); A. J. Buras and R. Fleischer, Eur. Phys. J. C **11**, 93 (1999). M. Gronau, D. Pirjol and T. M. Yan, Phys. Rev. D **60**, 034021 (1999); [Erratum-ibid. D **69**, 119901 (2004)].
- [17] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).
- [18] C. W. Chiang, M. Gronau, Z. Luo, J. L. Rosner and D. A. Suprun, Phys. Rev. D **69**, 034001 (2004); (ibid) arXiv:hep-ph/0404073; A. Ali, E. Lunghi and A. Y. Parkhomenko, Eur. Phys. J. C **36**, 183 (2004); J. Charles *et al.* [CKMfitter Group Collaboration], arXiv:hep-ph/0406184.
- [19] R. J. Hill, T. Becher, S. J. Lee and M. Neubert, JHEP **0407**, 081 (2004)
- [20] C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, arXiv:hep-ph/0502094.
- [21] Y. Grossman and H. R. Quinn, Phys. Rev. D **58**, 017504 (1998); J. Charles, Phys. Rev. D **59**, 054007 (1999);
- [22] M. Gronau, D. London, N. Sinha and R. Sinha, Phys. Lett. B **514**, 315 (2001).
- [23] D. Pirjol, Phys. Rev. D **60**, 054020 (1999); R. Fleischer, Phys. Lett. B **459**, 306 (1999).
- [24] G. Buchalla and A. S. Safir, Phys. Rev. Lett. **93**, 021801 (2004).

- [25] P. Ball and R. Zwicky, JHEP **0110**, 019 (2001); P. Ball and R. Zwicky, arXiv:hep-ph/0406232.
- [26] Z. Ligeti, arXiv:hep-ph/0408267.
- [27] C. W. Bauer, I. Z. Rothstein and I. W. Stewart, arXiv:hep-ph/0412120.
- [28] M. Gronau, E. Lunghi and D. Wyler, Phys. Lett. B **606**, 95 (2005) [arXiv:hep-ph/0410170].